Instituto Politécnico de Tomar

International students

MATHEMATICS TEST MODEL

Duration: 120 minutes

THE TEST IS FORMED BY 4 GROUPS (I, II, III AND IV) AND EACH GROUP IS FORMED BY TWO QUESTIONS (1.1, 1.2, ...). YOU SHOULD ONLY ANSWER TO **ONE QUESTION OF EACH GROUP** AND YOU MUST IDENTIFY CLEARLY THE QUESTIONS YOU CHOSE TO ANSWER.
EACH QUESTION IS WORTH 5 POINTS AND THE CLASSIFICATION OF THE TEST IS 20 POINTS.
JUSTIFY ALL YOUR ANSWERS AND SHOW **ALL THE CALCULATIONS** YOU MAKE.
ANY ATTEMPT OF FRAUD WILL CANCEL THE TEST.

I

1.1. Let A be the solution set, in $\mathbb{R}$, of inequality $\frac{x-9}{2}+5 \geq x^2$. It is known that $A \cap B =]0,1]$.
Which of the following sets could be set B? Justify.

(A) $]0,2]$ (B) $]-1,1[$ (C) $\mathbb{R}^-$ D) $\mathbb{Z}$

1.2. Consider three different natural numbers, of which 1 is the smallest. It is known that the exact value of the arithmetic mean of these three numbers is 3 and that their product is 15. What are the two natural numbers that satisfy these conditions?

II

2.1. Figure 1 shows part of the graph that represents a polynomial function f of degree 3.

It is known that:

- -2, 2 e 5 are zeros of f
- f' represents the derivative of f

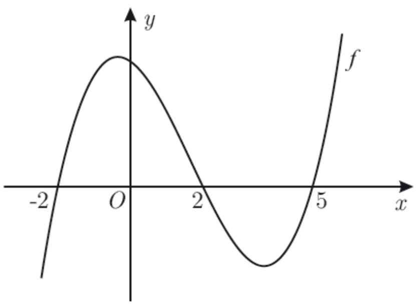

Figure 1

Indicate which of the following statements is true. Justification is needed.

(A) $f'(6) < 0$ (B) $f'(1) > 0$

(C) $f'(1) \times f'(6) > 0$ (D) $f'(-3) \times f'(6) > 0$

2.2. Figure 2 shows part of the graph that represents a polynomial function f of degree 3. Let $g(x) = x\,f(x-2)$. Say if each one of the following statements is true or false and justify answer:

(A) $D_g = \mathbb{R}\setminus\{2\}$

(B) If $x \in]-2,0[$, then $g(x) > 0$

(C) The function g has more zeros than the function f

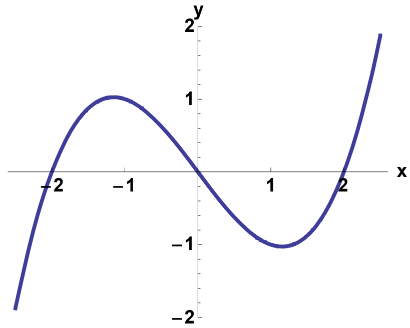

Figura 2

III

3.1. What is the solution set of the following inequality $8^{5x^2-43} - 64 > 0$.

3.2. Let a and b be two real numbers such that $1 < a < b$ and $\log_a b = \frac{1}{2}$. What is the value of $\log_a\left(\sqrt{a} \times b^3\right)$. Justify.

(A) 8 (B) 6 (C) 2 (D) −2

IV

4.1. You have 6 indistinguishable balls and 4 boxes. Without any of the boxes being empty, how many combinations are there of balls in boxes? How do you find the answer?

(A) 4 (B) 8 (C) 10 (D) 12

4.2. In the launch of two balanced data in the air, what is the probability that the total of the points is equal to 10? How do you find the answer?

(A) $\frac{1}{10}$ (B) $\frac{1}{12}$ (C) $\frac{1}{6}$ (D) $\frac{2}{23}$

FORM:

Lines

$$y - y_0 = \frac{y_1 - y_0}{x_1 - x_0}\left(x - x_0\right) \qquad y - f(x_0) = f'(x_0)\left(x - x_0\right)$$

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad V = \left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right) \qquad \Delta = b^2 - 4ac$$

Combinatorial Analysis

$$^{n}C_k = \binom{n}{k} = \frac{n!}{k!(n-k)!} \qquad ^{n}A_k = \frac{n!}{(n-k)!} \qquad P_n = n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$$

Newton's binomial

$$\left(a + b\right)^n = \sum_{p=0}^{n} {}^{n}C_p . a^{n-p} . b^p$$

Probabilities

$$P\left(A \mid B\right) = \frac{P\left(A \cap B\right)}{P(B)}, \text{ where } P(B) \neq 0 \qquad P\left(A \cup B\right) = P(A) + P(B) - P(A \cap B)$$

$$P\left(\overline{A}\right) = 1 - P(A)$$

Exponential rules: let $x, y \in \mathbb{R}$ and $n \in \mathbb{N}$, then

$$a^x \times a^y = a^{x+y} \qquad \frac{a^x}{a^y} = a^{x-y}, \text{ com } a \neq 0 \qquad \left(a^x\right)^y = a^{xy} \qquad \sqrt[n]{a^x} = a^{\frac{x}{n}} \qquad \left(\sqrt[n]{a}\right)^n = a$$

Logarithmic rules: let $a, b \in \mathbb{R}^+ \setminus \{1\}$ and $k \in \mathbb{R}$, then

$$\log_a x = y \Leftrightarrow x = a^y, \text{ where } x > 0 \qquad \log_a\left(a^x\right) = x \qquad a^{\log_a x} = x \qquad \log_a x = \frac{\log_b x}{\log_b a}$$

$$\log_a(x.y) = \log_a x + \log_a y \qquad \log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y \qquad \log_a\left(x^k\right) = k \log_a x$$

Derivation rules

$c' = 0$

$(cf)' = cf'$

$(f.g)' = f'g + fg'$

$\left(f^p\right)' = pf^{p-1} f', \quad p \in \mathbb{Q}$

$\left(e^f\right)' = f'e^f$

$(\ln f)' = \frac{f'}{f}$

$\left(f^g\right)' = g' f^g \ln f + g f^{g-1} f'$

$x' = 1$

$(f \pm g)' = f' \pm g'$

$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

$\left(\sqrt[n]{f}\right)' = \frac{f'}{n\sqrt[n]{f^{n-1}}}, \quad n \in \mathbb{N}$

$\left(a^f\right)' = f'a^f \ln a, \quad a \in \mathbb{R}^+$

$(\log_a f)' = \frac{f'}{f \ln a}, \quad , a \in \mathbb{R}^+ \setminus \{1\}$