esgt.ipt Escola Superior de Gestão de Tomar
Instituto Politécnico de Tomar

International students

APPLIED MATHEMATICS FOR SOCIAL SCIENCES TEST MODEL

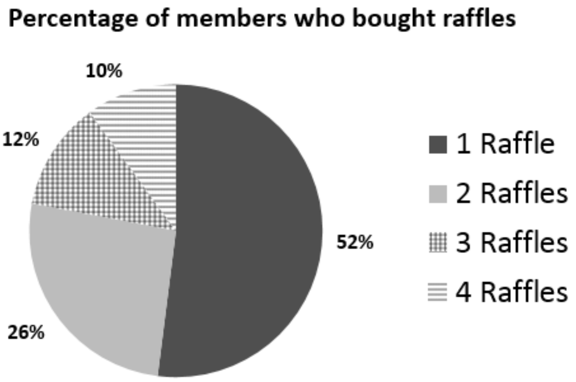
Duration: 120 minutes

THE TEST IS FORMED BY 4 GROUPS (I, II, III AND IV) AND EACH GROUP IS FORMED BY TWO QUESTIONS (1.1, 1.2, ...). YOU SHOULD ONLY ANSWER TO **ONE QUESTION OF EACH GROUP** AND YOU MUST IDENTIFY CLEARLY THE QUESTIONS YOU CHOSE TO ANSWER.
EACH QUESTION IS WORTH 5 POINTS AND THE CLASSIFICATION OF THE TEST IS 20 POINTS.
JUSTIFY ALL YOUR ANSWERS AND SHOW **ALL THE CALCULATIONS** YOU MAKE.
ANY ATTEMPT OF FRAUD WILL CANCEL THE TEST.

I

1.1. The number of raffle tickets sold to each member of a club varies between 1 to 4.

1.1.1. The following graph shows, among 50 members, the percentage of those who bought 1, 2, 3 or 4 raffles.



Determine the number of members, among these 50, who bought 2 raffles

1.1.2. A sample of 10 members was selected and a list was drawn which recorded the number of raffles bought by each of these 10 members. The median of this list of numbers is 2,5. Among these 10 members, it is known that four members bought 1 raffle, three members bought 3 raffles and one member bought 4 raffles. How many raffle tickets bought the two other members?

1.2. The variance of a sample of a statistical variable is 36. The sum of the squares of the deviations from the mean of that sample is 864. What is the sample size? How do you find the answer?

(A) 5 (B) 6 (C) 24 (D) 25

II

2.1. You have 6 indistinguishable balls and 4 boxes. Without any of the boxes being empty, how many combinations are there of balls in boxes? How do you find the answer?

(A) 4 (B) 8 (C) 10 (D) 12

2.2. In the launch of two balanced gaming dice in the air, what is the probability that the total of the points is equal to 10? How do you find the answer?

(A) $\frac{1}{10}$ (B) $\frac{1}{12}$ (C) $\frac{1}{6}$ (D) $\frac{2}{23}$

III

3.1. John bought a new car for 28800€. Considering that the devaluation of the car is 15% per year, how much the car will be worth in five years?

3.2. A financial institution, offers the guarantee of obtaining 1680€ for an initial deposit of 1500€, in simple interest regime, after six months, with a quarterly interest rate. According to this financial institution, the final capital is given by the following expression $C_n = C_0(1 + i \times n)$. Determine the annual simple interest rate of this deposit. Display the result as a percentage.

IV

4.1. Consider three different natural numbers, of which 1 is the smallest. It is known that the exact value of the arithmetic mean of these three numbers is 3 and that their product is 15. What are the two natural numbers that satisfy these conditions?

4.2. Let A be the solution set, in $\mathbb{R}$, of inequality $\frac{x-9}{2} + 5 \geq x^2$. Which of the following sets could be the set A? Justify and present all calculations.

(A) $\left]-\infty, -\frac{1}{2}\right] \cup [1, +\infty[$ (B) $]-\infty, -1] \cup \left[\frac{1}{2}, +\infty\right[$ (C) $\left[-\frac{1}{2}, 1\right]$ D) $\left[-1, \frac{1}{2}\right]$

FORM:

Quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \qquad V=\left(-\frac{b}{2a},-\frac{\Delta}{4a}\right) \qquad \Delta=b^2-4ac$$

Combinatorial Analysis

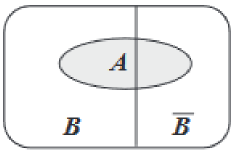
$${}^{n}C_k=\binom{n}{k}=\frac{n!}{k!(n-k)!} \qquad {}^{n}A_k=\frac{n!}{(n-k)!} \qquad P_n=n!=n\times(n-1)\times(n-2)\times\cdots\times3\times2\times1$$

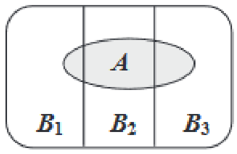
Probabilities

$$P(A|B)=\frac{P(A\cap B)}{P(B)},\ \text{where } P(B)\neq 0 \qquad P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$P(\bar{A})=1-P(A)$$

Total probability rule and Bayes' theorem





$$P(A)=P(A\cap B)+P(A\cap\bar{B})=$$
$$=P(B)\times P(A|B)+P(\bar{B})\times P(A|\bar{B})$$

$$P(B|A)=\frac{P(A\cap B)}{P(A)}=$$
$$=\frac{P(B)\times P(A|B)}{P(B)\times P(A|B)+P(\bar{B})\times P(A|\bar{B})}$$

$$P(A)=P(A\cap B_1)+P(A\cap B_2)+P(A\cap B_3)=$$
$$=P(B_1)\times P(A|B_1)+P(B_2)\times P(A|B_2)+P(B_3)\times P(A|B_3)$$

$$P(B_k|A)=\frac{P(A\cap B_k)}{P(A)}=$$
$$=\frac{P(B_k)\times P(A|B_k)}{P(B_1)\times P(A|B_1)+P(B_2)\times P(A|B_2)+P(B_3)\times P(A|B_3)}$$

where $k=1,2,3$

Newton's binomial

$$(a+b)^n=\sum_{p=0}^{n}{}^{n}C_p.a^{n-p}.b^p$$

Standard deviation

$$s=\sqrt{\frac{\sum_{i=1}^{n}n_i\left(x_i-\bar{x}\right)^2}{n-1}}\ \text{(sample)} \qquad \sigma=\sqrt{\frac{\sum_{i=1}^{n}n_i\left(x_i-\bar{x}\right)^2}{n}}\ \text{(population)}$$

Linear growth model

$$P_n=P_0+n\times r$$

Exponential growth model

$$P_n=P_0\times r^n$$

Simple Interest

$$C_n=C_0(1+i\times n)$$

Compound Interest

$$C_n=C_0\left(1+\frac{i}{100}\right)^n$$